LegendrePの性質と

$$\int_{-1}^{1} \frac{1}{\sqrt{1+t^2-2\,t\,x}}\ \frac{1}{\sqrt{1+s^2-2\,s\,x}}\,dx$$

LegendrePの定義

In[1]:= $\sum_{n=0}^{\infty} \text{LegendreP}[n,\ x]\ t^n$

Out[1]= $\frac{1}{\sqrt{1+t^2-2\,t\,x}}$

$\int_{-1}^{1} \frac{1}{\sqrt{1+t^2-2\,t\,x}}\ \frac{1}{\sqrt{1+s^2-2\,s\,x}}\,dx$ の結果

$\left(\sum_{n=0}^{\infty} \text{LegendreP}[n,\ x]\ t^n\right)\left(\sum_{n=0}^{\infty} \text{LegendreP}[n,\ x]\ s^n\right)$ を展開し、xを -1 から1まで積分

LegendreP[n, x] の直交性を考慮する

In[2]:= $\sum_{n=0}^{\infty} \frac{2}{2\,n+1}\ (t\ s)^n$

Out[2]= $\frac{2\,\text{ArcTanh}\left[\sqrt{s}\ \sqrt{t}\right]}{\sqrt{s}\ \sqrt{t}}$

In[4]:= $\frac{2\,\text{ArcTanh}\left[\sqrt{s}\ \sqrt{t}\right]}{\sqrt{s}\ \sqrt{t}}$ // TrigToExp

Out[4]= $-\frac{\text{Log}\left[1-\sqrt{s}\ \sqrt{t}\right]}{\sqrt{s}\ \sqrt{t}}+\frac{\text{Log}\left[1+\sqrt{s}\ \sqrt{t}\right]}{\sqrt{s}\ \sqrt{t}}$

LegendreP

In[8]:=
```
n =.;
For[n = 1, n ≤ 10, n++,
 Print["LegendreP[", n, ",x] = ", LegendreP[n, x]]]
```

LegendreP[1,x] = x

LegendreP[2,x] = $\frac{1}{2}\left(-1+3x^2\right)$

LegendreP[3,x] = $\frac{1}{2}\left(-3x+5x^3\right)$

LegendreP[4,x] = $\frac{1}{8}\left(3-30x^2+35x^4\right)$

LegendreP[5,x] = $\frac{1}{8}\left(15x-70x^3+63x^5\right)$

LegendreP[6,x] = $\frac{1}{16}\left(-5+105x^2-315x^4+231x^6\right)$

LegendreP[7,x] = $\frac{1}{16}\left(-35x+315x^3-693x^5+429x^7\right)$

LegendreP[8,x] = $\frac{1}{128}\left(35-1260x^2+6930x^4-12012x^6+6435x^8\right)$

LegendreP[9,x] = $\frac{1}{128}\left(315x-4620x^3+18018x^5-25740x^7+12155x^9\right)$

LegendreP[10,x] = $\frac{1}{256}\left(-63+3465x^2-30030x^4+90090x^6-109395x^8+46189x^{10}\right)$

LegendrePの直交性の計算

```
For[n = 1, n ≤ 5, n++,
 For[m = 1, m ≤ 5, m++,
  Print["n = ", n, " , m = ", m,
   " , integral = ", Integrate[LegendreP[n, x] LegendreP[m, x],
    {x, -1, 1}, Assumptions → Element[n, Integers]]]]]
```

n = 1 , m = 1 , integral = $\frac{2}{3}$

n = 1 , m = 2 , integral = 0

n = 1 , m = 3 , integral = 0

n = 1 , m = 4 , integral = 0

n = 1 , m = 5 , integral = 0

n = 2 , m = 1 , integral = 0

n = 2 , m = 2 , integral = $\frac{2}{5}$

n = 2 , m = 3 , integral = 0

n = 2 , m = 4 , integral = 0

n = 2 , m = 5 , integral = 0

n = 3 , m = 1 , integral = 0

n = 3 , m = 2 , integral = 0

n = 3 , m = 3 , integral = $\frac{2}{7}$

n = 3 , m = 4 , integral = 0

n = 3 , m = 5 , integral = 0

n = 4 , m = 1 , integral = 0

n = 4 , m = 2 , integral = 0

n = 4 , m = 3 , integral = 0

n = 4 , m = 4 , integral = $\frac{2}{9}$

n = 4 , m = 5 , integral = 0

n = 5 , m = 1 , integral = 0

n = 5 , m = 2 , integral = 0

n = 5 , m = 3 , integral = 0

n = 5 , m = 4 , integral = 0

n = 5 , m = 5 , integral = $\frac{2}{11}$