

$$\int \frac{1}{x^6+1} dx$$

x^6+1 の因数分解

`Clear[f]`

`Factor[x^6 + 1, Extension -> Sqrt[3]]`

$$-(-1 + \sqrt{3}x - x^2)(1 + x^2)(1 + \sqrt{3}x + x^2)$$

$\frac{1}{x^6+1}$ を最小の分母を使った項の和に書き換える.

$$f[x_] := \text{Apart}\left[\frac{1}{-(-1 + \sqrt{3}x - x^2)(1 + x^2)(1 + \sqrt{3}x + x^2)}\right]$$

`f[x]`

$$\frac{-2\sqrt{3} + 3x}{6\sqrt{3}(-1 + \sqrt{3}x - x^2)} + \frac{1}{3(1 + x^2)} + \frac{2 + \sqrt{3}x}{6(1 + \sqrt{3}x + x^2)}$$

このあと、各項に対して、不定積分をおこなうことになる。

`Length[f[x]]`

3

`f[x][[1]]`

$$\frac{-2\sqrt{3} + 3x}{6\sqrt{3}(-1 + \sqrt{3}x - x^2)}$$

`Integrate[f[x][[1]], x]`

$$-\frac{1}{6} \text{ArcTan}[\sqrt{3} - 2x] - \frac{\text{Log}[1 - \sqrt{3}x + x^2]}{4\sqrt{3}}$$

`Integrate[f[x][[2]], x]`

$$\frac{\text{ArcTan}[x]}{3}$$

`Integrate[f[x][[3]], x]`

$$\frac{1}{6} \text{ArcTan}[\sqrt{3} + 2x] + \frac{\text{Log}[1 + \sqrt{3}x + x^2]}{4\sqrt{3}}$$

$$\int \frac{1}{x^8+1} dx$$

`Clear[f]`

x^8+1 の因数分解

```
(x^2 + Sqrt[2 - Sqrt[2]] x + 1) (x^2 - Sqrt[2 - Sqrt[2]] x + 1)
(x^2 + Sqrt[2 + Sqrt[2]] x + 1) (x^2 - Sqrt[2 + Sqrt[2]] x + 1) // Expand
1 + x^8
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$\frac{1}{x^6 + 1}$ を最小の分母を使った項の和に書き換える。

```
f[x_] := 1 / ((x^2 + Sqrt[2 - Sqrt[2]] x + 1) (x^2 - Sqrt[2 - Sqrt[2]] x + 1)
(x^2 + Sqrt[2 + Sqrt[2]] x + 1) (x^2 - Sqrt[2 + Sqrt[2]] x + 1)) // Apart
```

f[x]

$$\frac{-2\sqrt{2-\sqrt{2}} + 2x - \sqrt{2}x}{8\sqrt{2-\sqrt{2}}(-1+\sqrt{2-\sqrt{2}}x-x^2)} + \frac{2\sqrt{2-\sqrt{2}} - 2\sqrt{2(2-\sqrt{2})} + 4x - 3\sqrt{2}x}{4\sqrt{2(2-\sqrt{2})}(-2+\sqrt{2})(1+\sqrt{2-\sqrt{2}}x+x^2)} +$$

$$\frac{(-1+\sqrt{2})(-2\sqrt{2+\sqrt{2}} + 2x + \sqrt{2}x)}{4(-2+\sqrt{2})\sqrt{2(2+\sqrt{2})}(1-\sqrt{2+\sqrt{2}}x+x^2)} +$$

$$\frac{-2\sqrt{2+\sqrt{2}} + \sqrt{2(2+\sqrt{2})} - x}{4(-2+\sqrt{2})\sqrt{2+\sqrt{2}}(1+\sqrt{2+\sqrt{2}}x+x^2)}$$

このあと、各項に対して、不定積分をおこなうことになる。

Length[f[x]]

4

Integrate[f[x][[1]], x]

$$\frac{1}{8}\sqrt{2+\sqrt{2}}\text{ArcTan}\left[\frac{-\sqrt{2-\sqrt{2}}+2x}{\sqrt{2+\sqrt{2}}}\right] - \frac{1}{16}\sqrt{2-\sqrt{2}}\text{Log}\left[1-\sqrt{2-\sqrt{2}}x+x^2\right]$$

Integrate[f[x][[2]], x]

$$\frac{-\sqrt{6-4\sqrt{2}}\text{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}}+2x}{\sqrt{2+\sqrt{2}}}\right] + \frac{1}{2}(4-3\sqrt{2})\text{Log}\left[1+\sqrt{2-\sqrt{2}}x+x^2\right]}{4\sqrt{4-2\sqrt{2}}(-2+\sqrt{2})}$$

Integrate[f[x][[3]], x]

$$\left((-1 + \sqrt{2}) \left((-2 + \sqrt{2}) \sqrt{3 + 2\sqrt{2}} \operatorname{ArcTan}\left[\frac{-\sqrt{2 + \sqrt{2}} + 2x}{\sqrt{2 - \sqrt{2}}}\right] + \frac{1}{2} (2 + \sqrt{2}) \operatorname{Log}\left[1 - \sqrt{2 + \sqrt{2}} x + x^2\right]\right) \right) / \left(4 (-2 + \sqrt{2}) \sqrt{2(2 + \sqrt{2})} \right)$$

Integrate[f[x][[4]], x]

$$\left((-3 + 2\sqrt{2}) \sqrt{3 + 2\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2 + \sqrt{2}} + 2x}{\sqrt{2 - \sqrt{2}}}\right] - \frac{1}{2} \operatorname{Log}\left[1 + \sqrt{2 + \sqrt{2}} x + x^2\right] \right) / \left(4 (-2 + \sqrt{2}) \sqrt{2 + \sqrt{2}} \right)$$

$x^6 + 1$ の因数分解 (Mathematicaで行う)

これだと、筆算をするきはなくなってしまふ。

Factor $[1 + x^8, \text{Extension} \rightarrow \{\sqrt{2}, \sqrt{2(2 + \sqrt{2})}, \sqrt{2 - \sqrt{2}}\}]$

$$\frac{1}{10899829617121} \left(-1817 + \left(-1389 \sqrt{2 - \sqrt{2}} + 441 \sqrt{2(2 - \sqrt{2})} - 454 \sqrt{2 + \sqrt{2}} + 13 \sqrt{2(2 + \sqrt{2})} \right) x - 1817 x^2 \right) \left(-1817 + \left(447 \sqrt{2 - \sqrt{2}} + 1212 \sqrt{2(2 - \sqrt{2})} - 160 \sqrt{2 + \sqrt{2}} + 765 \sqrt{2(2 + \sqrt{2})} \right) x - 1817 x^2 \right) \left(1817 + \left(-1389 \sqrt{2 - \sqrt{2}} + 441 \sqrt{2(2 - \sqrt{2})} - 454 \sqrt{2 + \sqrt{2}} + 13 \sqrt{2(2 + \sqrt{2})} \right) x + 1817 x^2 \right) \left(1817 + \left(447 \sqrt{2 - \sqrt{2}} + 1212 \sqrt{2(2 - \sqrt{2})} - 160 \sqrt{2 + \sqrt{2}} + 765 \sqrt{2(2 + \sqrt{2})} \right) x + 1817 x^2 \right)$$

$$\begin{aligned}
& 1 / \left(\frac{1}{10\,899\,829\,617\,121} \right. \\
& \left. \left(-1817 + \left(-1389 \sqrt{2 - \sqrt{2}} + 441 \sqrt{2(2 - \sqrt{2})} - 454 \sqrt{2 + \sqrt{2}} + 13 \sqrt{2(2 + \sqrt{2})} \right) x - \right. \right. \\
& \left. \left. 1817 x^2 \right) \right. \\
& \left. \left(-1817 + \left(447 \sqrt{2 - \sqrt{2}} + 1212 \sqrt{2(2 - \sqrt{2})} - 160 \sqrt{2 + \sqrt{2}} + 765 \sqrt{2(2 + \sqrt{2})} \right) x - \right. \right. \\
& \left. \left. 1817 x^2 \right) \left(1817 + \left(-1389 \sqrt{2 - \sqrt{2}} + 441 \sqrt{2(2 - \sqrt{2})} - \right. \right. \right. \\
& \left. \left. \left. 454 \sqrt{2 + \sqrt{2}} + 13 \sqrt{2(2 + \sqrt{2})} \right) x + 1817 x^2 \right) \right. \\
& \left. \left(1817 + \left(447 \sqrt{2 - \sqrt{2}} + 1212 \sqrt{2(2 - \sqrt{2})} - 160 \sqrt{2 + \sqrt{2}} + 765 \sqrt{2(2 + \sqrt{2})} \right) x + \right. \right. \\
& \left. \left. 1817 x^2 \right) \right) // \text{Apart} \\
& \left(1817 \left(-193\,574\,936\,597\,605\,999\,804 + 121\,974\,935\,718\,226\,684\,034 \sqrt{2} - \right. \right. \\
& \left. \left. 50\,003\,600\,552\,628\,297\,696 \sqrt{(2 - \sqrt{2})(2 + \sqrt{2})} + 24\,816\,133\,133\,204\,613\,564 \right. \right. \\
& \left. \left. \sqrt{2(2 - \sqrt{2})(2 + \sqrt{2})} - 74\,930\,701\,072\,074\,138\,960 \sqrt{2 - \sqrt{2}} x + \right. \right. \\
& \left. \left. 45\,517\,490\,681\,139\,008\,271 \sqrt{2(2 - \sqrt{2})} x - 22\,023\,012\,103\,155\,382\,826 \sqrt{2 + \sqrt{2}} x + \right. \right. \\
& \left. \left. 12\,491\,189\,004\,815\,567\,724 \sqrt{2(2 + \sqrt{2})} x \right) \right) / \\
& \left(8 \left(53\,581\,794\,256\,688\,065\,094 - 30\,435\,169\,874\,435\,498\,425 \sqrt{2} - 5\,550\,497\,708\,363\,694\,744 \right. \right. \\
& \left. \left. \sqrt{(2 - \sqrt{2})(2 + \sqrt{2})} + 9\,194\,770\,454\,455\,160\,622 \sqrt{2(2 - \sqrt{2})(2 + \sqrt{2})} \right) \right. \\
& \left. \left(-1817 - 1389 \sqrt{2 - \sqrt{2}} x + 441 \sqrt{2(2 - \sqrt{2})} x - 454 \sqrt{2 + \sqrt{2}} x + \right. \right. \\
& \left. \left. 13 \sqrt{2(2 + \sqrt{2})} x - 1817 x^2 \right) \right) + \\
& \left(1817 \left(-6\,756\,011\,301\,096\,606\,546\,146\,887\,157\,504\,132 + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 4\,939\,691\,557\,184\,346\,760\,507\,605\,005\,800\,798 \sqrt{2} - \\
& 5\,724\,166\,847\,806\,078\,182\,996\,982\,983\,093\,696 \sqrt{(2-\sqrt{2})(2+\sqrt{2})} + \\
& 4\,162\,480\,941\,170\,034\,695\,562\,821\,556\,044\,964 \sqrt{2(2-\sqrt{2})(2+\sqrt{2})} - \\
& 4\,010\,024\,558\,690\,730\,873\,886\,823\,597\,982\,762 \sqrt{2-\sqrt{2}} x + \\
& 2\,639\,396\,736\,598\,305\,702\,493\,368\,125\,744\,307 \sqrt{2(2-\sqrt{2})} x - \\
& 10\,073\,293\,322\,509\,073\,701\,362\,937\,826\,764\,274 \sqrt{2+\sqrt{2}} x + \\
& 7\,041\,658\,940\,599\,902\,287\,624\,880\,712\,373\,518 \sqrt{2(2+\sqrt{2})} x \Big) \Big) / \\
& \left(8 \left(-42\,253\,677\,190\,321 + 26\,024\,183\,733\,912 \sqrt{2} - 26\,024\,183\,733\,912 \right. \right. \\
& \quad \left. \left. \sqrt{(2-\sqrt{2})(2+\sqrt{2})} + 15\,676\,923\,786\,600 \sqrt{2(2-\sqrt{2})(2+\sqrt{2})} \right) \right. \\
& \left(53\,581\,794\,256\,688\,065\,094 - 30\,435\,169\,874\,435\,498\,425 \sqrt{2} - 5\,550\,497\,708\,363\,694\,744 \right. \\
& \quad \left. \left. \sqrt{(2-\sqrt{2})(2+\sqrt{2})} + 9\,194\,770\,454\,455\,160\,622 \sqrt{2(2-\sqrt{2})(2+\sqrt{2})} \right) \right. \\
& \left(-1817 + 447 \sqrt{2-\sqrt{2}} x + 1212 \sqrt{2(2-\sqrt{2})} x - 160 \sqrt{2+\sqrt{2}} x + \right. \\
& \quad \left. \left. 765 \sqrt{2(2+\sqrt{2})} x - 1817 x^2 \right) \right) - \\
& \left(1817 \left(4\,533\,986\,184\,511\,928\,644\,209\,889\,232\,057\,908\,122 \sqrt{2-\sqrt{2}} - \right. \right. \\
& \quad 2\,702\,060\,342\,681\,741\,882\,955\,478\,161\,994\,248\,906 \sqrt{2(2-\sqrt{2})} + \\
& \quad 8\,512\,715\,266\,815\,726\,415\,457\,645\,771\,305\,210\,268 \sqrt{2+\sqrt{2}} - \\
& \quad 5\,810\,654\,924\,133\,984\,532\,502\,167\,609\,310\,961\,362 \sqrt{2(2+\sqrt{2})} - \\
& \quad 4\,889\,964\,297\,236\,158\,335\,818\,877\,910\,843\,929\,646 x + \\
& \quad 3\,310\,122\,780\,940\,447\,324\,006\,761\,387\,386\,100\,545 \sqrt{2} x - \\
& \quad 2\,597\,426\,979\,410\,604\,326\,675\,161\,495\,015\,502\,712 \sqrt{(2-\sqrt{2})(2+\sqrt{2})} x + \\
& \quad \left. \left. 1\,732\,286\,347\,088\,236\,170\,577\,839\,063\,051\,366\,990 \sqrt{2(2-\sqrt{2})(2+\sqrt{2})} x \right) \right) \Big) /
\end{aligned}$$

$$\begin{aligned}
& \left(4 \left(1836 \sqrt{2 - \sqrt{2}} + 771 \sqrt{2(2 - \sqrt{2})} + 294 \sqrt{2 + \sqrt{2}} + 752 \sqrt{2(2 + \sqrt{2})} \right) \right. \\
& \left(-42\,253\,677\,190\,321 + 26\,024\,183\,733\,912 \sqrt{2} - 26\,024\,183\,733\,912 \right. \\
& \quad \left. \sqrt{(2 - \sqrt{2})(2 + \sqrt{2})} + 15\,676\,923\,786\,600 \sqrt{2(2 - \sqrt{2})(2 + \sqrt{2})} \right) \\
& \left(53\,581\,794\,256\,688\,065\,094 - 30\,435\,169\,874\,435\,498\,425 \sqrt{2} - 5\,550\,497\,708\,363\,694\,744 \right. \\
& \quad \left. \sqrt{(2 - \sqrt{2})(2 + \sqrt{2})} + 9\,194\,770\,454\,455\,160\,622 \sqrt{2(2 - \sqrt{2})(2 + \sqrt{2})} \right) \\
& \left(1817 - 1389 \sqrt{2 - \sqrt{2}} x + 441 \sqrt{2(2 - \sqrt{2})} x - 454 \sqrt{2 + \sqrt{2}} x + \right. \\
& \quad \left. 13 \sqrt{2(2 + \sqrt{2})} x + 1817 x^2 \right) \Bigg) + \\
& \left(1817 \left(896\,853\,974\,398\,988\,853\,804\,369\,993\,663\,538\,662 \sqrt{2 - \sqrt{2}} - \right. \right. \\
& \quad 1\,138\,123\,561\,208\,680\,507\,650\,780\,041\,929\,089\,318 \sqrt{2(2 - \sqrt{2})} + \\
& \quad 4\,598\,492\,699\,876\,035\,863\,769\,129\,862\,262\,912\,964 \sqrt{2 + \sqrt{2}} - \\
& \quad 3\,460\,369\,138\,667\,355\,356\,118\,349\,820\,333\,823\,646 \sqrt{2(2 + \sqrt{2})} - \\
& \quad 2\,922\,420\,835\,711\,397\,035\,698\,614\,806\,768\,313\,028 x + \\
& \quad 1\,710\,115\,689\,536\,573\,546\,947\,144\,241\,702\,770\,471 \sqrt{2} x - \\
& \quad 2\,422\,811\,491\,066\,416\,544\,278\,744\,134\,073\,368\,304 \sqrt{(2 - \sqrt{2})(2 + \sqrt{2})} x + \\
& \quad \left. \left. 1\,461\,210\,417\,855\,698\,517\,849\,307\,403\,384\,156\,514 \sqrt{2(2 - \sqrt{2})(2 + \sqrt{2})} x \right) \right) \Bigg) / \\
& \left(4 \left(1836 \sqrt{2 - \sqrt{2}} + 771 \sqrt{2(2 - \sqrt{2})} + 294 \sqrt{2 + \sqrt{2}} + 752 \sqrt{2(2 + \sqrt{2})} \right) \right. \\
& \left(-42\,253\,677\,190\,321 + 26\,024\,183\,733\,912 \sqrt{2} - 26\,024\,183\,733\,912 \right. \\
& \quad \left. \sqrt{(2 - \sqrt{2})(2 + \sqrt{2})} + 15\,676\,923\,786\,600 \sqrt{2(2 - \sqrt{2})(2 + \sqrt{2})} \right) \\
& \left(53\,581\,794\,256\,688\,065\,094 - 30\,435\,169\,874\,435\,498\,425 \sqrt{2} - 5\,550\,497\,708\,363\,694\,744 \right.
\end{aligned}$$

$$\sqrt{(2 - \sqrt{2})(2 + \sqrt{2})} + 9194770454455160622 \sqrt{2(2 - \sqrt{2})(2 + \sqrt{2})}$$

$$\left(1817 + 447 \sqrt{2 - \sqrt{2}} x + 1212 \sqrt{2(2 - \sqrt{2})} x - 160 \sqrt{2 + \sqrt{2}} x + \right.$$

$$\left. 765 \sqrt{2(2 + \sqrt{2})} x + 1817 x^2 \right)$$