アステロイドの体積

定義と描画

定義

アステロイドの曲面の式は、
astell[u_,v_]:=a{Sin[u]Cos[v],Sin[u]Sin[v],Cos[u]}^3

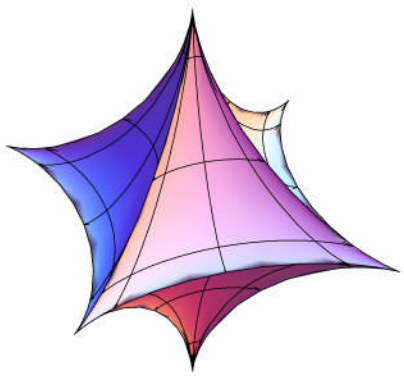
```
astell[u_, v_, a_] := a {Sin[u] Cos[v], Sin[u] Sin[v], Cos[u]}^3
```

描画

```
u = .
```

```
v = .
```

```
ParametricPlot3D[astell[u, v, 1], {u, 0, 2 Pi},
 {v, 0, 2 Pi}, Axes → None, Boxed → False, PlotRange → All]
```



Jacobianの算出

```
J[u_, v_, a_] := {D[astell[u, v, a], u], D[astell[u, v, a], v], astell[u, v, a]}
```

```
J[u, v, a] // MatrixForm
```

$$\begin{pmatrix} 3\,a\,\text{Cos}[u]\,\text{Cos}[v]^3\,\text{Sin}[u]^2 & 3\,a\,\text{Cos}[u]\,\text{Sin}[u]^2\,\text{Sin}[v]^3 & -3\,a\,\text{Cos}[u]^2\,\text{Sin}[u] \\ -3\,a\,\text{Cos}[v]^2\,\text{Sin}[u]^3\,\text{Sin}[v] & 3\,a\,\text{Cos}[v]\,\text{Sin}[u]^3\,\text{Sin}[v]^2 & 0 \\ a\,\text{Cos}[v]^3\,\text{Sin}[u]^3 & a\,\text{Sin}[u]^3\,\text{Sin}[v]^3 & a\,\text{Cos}[u]^3 \end{pmatrix}$$

```
Det[J[u, v, a]]
```

$$9\,a^3\,\text{Cos}[u]\,\text{Cos}[v]\,\text{Sin}[u]^4\,\text{Sin}[v]\,(\text{Cos}[u]^3\,\text{Cos}[v]^3\,\text{Sin}[u]\,\text{Sin}[v] + \text{Cos}[u]\,\text{Cos}[v]^3\,\text{Sin}[u]^3\,\text{Sin}[v] + \text{Cos}[u]^3\,\text{Cos}[v]\,\text{Sin}[u]\,\text{Sin}[v]^3 + \text{Cos}[u]\,\text{Cos}[v]\,\text{Sin}[u]^3\,\text{Sin}[v]^3)$$

```
Det[J[u, v, a]] // Simplify // InputForm
```

```
9*a^3*Cos[u]^2*Cos[v]^2*Sin[u]^5*Sin[v]^2
```

```
dj[u_, v_, a_] := Det[J[u, v, a]] // Simplify
```

```
dj[u, v, a]
```

$$9\,a^3\,\text{Cos}[u]^2\,\text{Cos}[v]^2\,\text{Sin}[u]^5\,\text{Sin}[v]^2$$

```
Integrate[9 a^3 Cos[u]^2 Cos[v]^2 Sin[u]^5 Sin[v]^2, {u, 0, Pi / 2}]
```

$$\frac{6}{35}\,a^3\,\text{Sin}[2\,v]^2$$

```
Integrate[6/35 a^3 Sin[2 v]^2, {v, 0, Pi / 2}]
```

$$\frac{3\,a^3\,\pi}{70}$$

```
Integrate[3 a^3 π/70, a] // Simplify
```

$$\frac{3\,a^4\,\pi}{280}$$

```
4 * Integrate[3 a^3 π/70, a] // Simplify
```

$$\frac{3\,a^4\,\pi}{70}$$